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THE PENETRATION OF STAR-SHAPED BODIES INTO A COMPRESSIBLE FLUID*

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A linear formulation and the method of superposition of special solutions are used to obtain exact analytic solutions of non-stationary spatial problems of the penetration of star-shaped bodies with an arbitrary number of lobes or fins, into a compressible fluid, at subsonic and supersonic speeds.

A general formulation of this problem for a bundle of wedges was first given in /1/.

1. Formulation of the problem of the penetration of a star-shaped body into a compressible fluid. We shall consider the normal entry of a star-shaped solid S into a liquid half-space (Fig.1). The solid S consists of m identical thin fins or blades, distributed symmetrically along a circle, and the thickness of a fin is small compared with its longitudinal and transverse dimensions. The velocity of motion of the body $v(t)$ can be either subsonic, or supersonic. Thanks to the presence of a small parameter, namely the relative thickness of the body, the problem can be linearized, and its solution for the velocity potential of perturbed motion $\varphi(t, x, y, z)$ in a fixed Cartesian x, y, z-coordinate system (in which the plane $x=0$ coincides with the free surface of the unperturbed liquid and the x axis is directed along the axis of the body), will be described by the following wave equation:

$$\partial_{xx}\varphi + \partial_{yy}\varphi + \partial_{zz}\varphi = c_0^{-2}\partial_{tt}\varphi \qquad (x > 0,\ t > 0) \tag{1.1}$$

with zero initial conditions

$$\varphi = \partial_t\varphi = 0 \qquad (t = 0) \tag{1.2}$$

and the following boundary conditions:

$$\varphi = 0\ (x = 0) \tag{1.3}$$

$$\partial_n\varphi = v_n(t, x, y, z) \quad \text{on } S_k \tag{1.4}$$

Here c_0 is the speed of sound in the unperturbed liquid and S_k is the projection of the k-th fin on its plane of symmetry onto which the boundary conditions are carried by virtue of the linear formulation of the problem. Above and henceforth, $k = 1, 2, \ldots, m$. We note that since the fins are symmetrical and are symmetrically distributed in space, it follows that the condition $\partial_n\varphi = 0$ must hold on the planes of symmetry of the fins outside their projections S_k (Fig.2).

Fig.2 shows the section of three-dimensional space by the plane $x = \text{const} > 0$ after the linearization of the problem. The dashed lines represent the parts of the cross-sections of planes at whose points the condition $\partial_n\varphi = 0$ holds, and the corresponding solid lines O_1A_1, O_1A_2, ..., O_1A_m represent the cross-sections of the corresponding projections S_1, S_2, ..., S_m, on each side of which the symmetrical condition $\partial_n\varphi = v_n(t, x, y, z)$ holds.

In order to ensure that the solution of the problem in question is unique, it is sufficient to require /2/ that the pressure, in the neighbourhood of the lines and points at which the boundary of the body is not smooth, satisfies the following constraints: $p = C + O(\varepsilon^\beta)$ as $\varepsilon \to 0$, where $\beta > -1$ for a moving point (the sharp bodies and corner points of the moving edges of projections S_k), $\beta > -1/2$ for a moving line (points on the moving edges S_k without the corner points) and for the fixed point O, $\beta > 0$ for the fixed line (the line of intersection of projections S_k which is a part of the x axis and the line of intersection of the projections S_1, S_2, ..., S_m with the plane $x = 0$).

Here ε is the distance to the line (point) at which the body is no longer smooth and the quantity C and the residue term $O(\varepsilon^\beta)$ can depend on time and other spatial variables.

We note that the pressure is determined with help of the linearized formula for the Cauchy-Lagrange integral

$$p - p_0 = -\rho_0\partial_t\varphi \tag{1.5}$$

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where p_0 and ρ_0 is the pressure and density of the unperturbed liquid.

For the flow pattern discussed above (Fig.2) it follows that the flow within every sector with the opening angle $\psi_0 = 2\pi/m$, e.g., $A_1'A_1O_1A_2A_2'$, is the same as in the remaining sectors.

Thus the solution of the initial problem of penetration is reduced to that of solving Eq.(1.1) within the spatial sector $A_1'A_1O_1A_2A_2'$ in the region $x > 0$ (Fig.2), with zero initial conditions (1.2), boundary conditions (1.3), a given normal derivatives $\partial_n\varphi$ on the boundary planes, $\partial_n\varphi = v_n$ on O_1A_1 and O_1A_2, $\partial_n\varphi = 0$ on A_1A_1' and A_2A_2', and with the constraints shown above for the neighbourhoods of the breaks in the surface of the body.

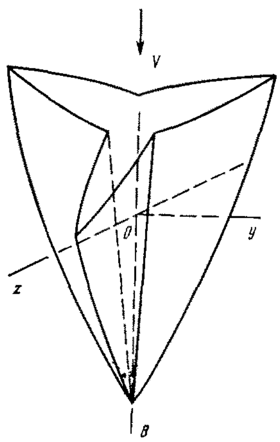

Fig.1

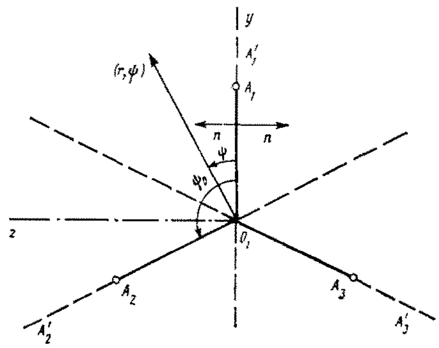

Fig.2

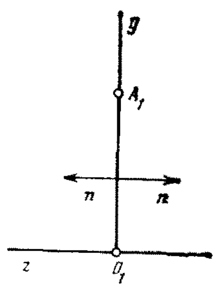

Fig.3

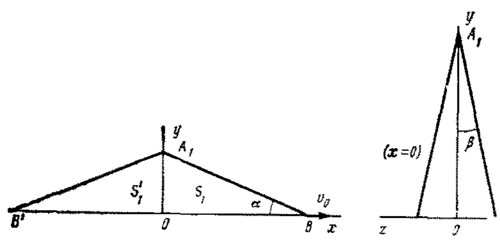

Fig.4

2. *Method of constructing the solution of the problem for an arbitrary number of fins.* In order to construct the solution of the problem formulated above for the sector $A_1'A_1O_1A_2A_2'$ with the aperture angle $\psi_0 = 2\pi/m$, we shall use the principle of superposition of solutions of the linear problems. We shall construct a solution of the problem in the form of a linear combination of the auxiliary solutions of certain mathematical problems each of which may, generally speaking, have no physical meaning.

Let a solution $\varphi = \varphi_1(t, x, y, z)$ of wave Eq.(1.1) for the half-space $x > 0$ (see the cross-section $x = \text{const} > 0$ in Fig.3) satisfy the zero initial conditions (1.2), the boundary conditions (1.3) and conditions in the plane $z = 0$; $\partial_n\varphi = v_n(t, x, y, \pm 0)$ on S_1 (O_1A_1 in Fig.3, $\partial_n\varphi = 0$ outside S_1 (the y axis outside the segment O_1A_1 in Fig.3).

We shall first consider the case when $m = 2l - 1$ (the star-shaped body has an odd number of fins). Let us superimpose on the solution $\varphi = \varphi_1(t, x, y, z)$ the solutions of the problem obtained by consecutive rotation of the plane $z = 0$ (Fig.3) about the x axis, with the normal derivative $\partial_n\varphi$ specified on it, by the angles $\psi_k = \psi_0 k$ $(k = 1, 2, \ldots, 2l - 2)$, so that the cross-section O_1A_1 will occupy consectively the positions $O_1A_2, O_1A_3, \ldots, O_1A_{2l-1}$. Since the cross-sections $O_1A_2, O_1A_3, \ldots, O_1A_{2l-1}$ are distributed symmetrically (and their even number is $2l - 2$) about the plane $z = 0$, it follows that the additional solutions will not affect the values of the derivative $\partial_n\varphi$ in this plane. Similarly, the values of $\partial_n\varphi$ in the rotated planes will also not be affected. As a result, we obtain the solution of the problem periodic with respect to the angle ψ $(y = r\cos\psi,\ z = r\sin\psi)$ with period $\psi = \psi_0$ corresponding to the configuration in Fig.2, i.e. we obtain a solution of the initial linear Problem (1.1)-(1.5) which has the form

$$\varphi(t, x, y, z) = \sum_{k=1}^{m} \varphi_1\left(t, x, r, \psi - \frac{2\pi(k-1)}{m}\right) \tag{2.1}$$

$$\varphi_1(t, x, r, \psi) \equiv \varphi_1(t, x, y, z), \quad x > 0, \quad (y = r\cos\psi, \; z = r\sin\psi)$$

Formula (2.1) is obtained for a body with an odd number of fins. In the case of an even number of fins the construction is analogous and the solution is also given by formula (2.1).

Thus in order to obtain the solution (2.1) we must obtain an auxiliary solution $\varphi = \varphi_1(t, x, y, z)$ in the region $x \geqslant 0$. To satisfy the condition $\varphi = 0$ in the plane $x = 0$, we shall continue the solution $\varphi = \varphi_1$ in an odd manner from the region $x > 0$ to the region $x < 0$.

Further, taking into account the fact that φ_1 is even in z, we finally arrive at the problem of determining the function $\varphi_1(t, x, y, z)$, satisfying, in the region $z > 0$, the wave Eq.(1.1), the zero initial conditions (1.2) and the following boundary condition in the plane $z = 0$:

$$\partial_z\varphi_1(t, x, y, z)|_{z=+0} = v_n(t, x, y)\operatorname{sgn} xH[F(t, x, y)]H(t) \tag{2.2}$$

$$(H(x) = 1, \; x > 0; \; H(x) = 0, \; x < 0)$$

Here $F(t, x, y) = 0$ is the equation of the line $BA_1B'OB$ at $t > 0$ (Fig.4) bounding the region S_1 and its symmetrical image S_1', and the inequality $F(t, x, y) > 0$ holds at points inside the region $S_1 + S_1'$.

The solution of this problem is known (it can be obtained directly using integral transforms) and has the form /3/

$$\varphi_1(t, x, y, z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\partial_{z_0}\varphi_1\left(t - \frac{R}{c_0}, \xi, \eta, z_0\right)\Big|_{z_0=+0}\frac{d\xi\, d\eta}{R} \tag{2.3}$$

$$R = [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}$$

(the quantity $\partial_{z_0}\varphi_1|_{z_0=+0}$ is defined in (2.2)).

Since the solution obtained is even in z, it can be used over the whole space $-\infty < z < \infty$, and it is continuous in it. However, and passing through the plane region $S_1 + S_1'$ the normal derivative becomes discontinuous (or, according to (2.2)),

$$(\partial_z\varphi_1)_{z=+0} = -(\partial_z\varphi_1)_{z=-0} = \partial_n\varphi_1 = v_n \operatorname{sgn} x)$$

Taking into account the relations (1.5), (2.1) and (2.3), we obtain

$$p(t, x, r, \psi) - p_0 = \sum_{k=1}^{m} p_1\left(t, x, r, \psi - \frac{2\pi(k-1)}{m}\right)$$

$$p_1(t, x, r, \psi) = -\rho_0\partial_t\varphi_1(t, x, r, \psi) \tag{2.4}$$

In particular, if the form and the cross-section of the fin are both triangular (Fig.4) and the rate of penetration of the body is constant $(v(t) \equiv v_0)$, then $v_n(t, x, y) = v_0\gamma$, where $\gamma = (1 + \operatorname{ctg}^2\alpha \operatorname{cosec}^2\beta)^{-1/2} \approx \beta \operatorname{tg}\alpha$ (or $\beta \ll 1$), and

$$F(t, x, y) = \left(1 - \frac{|x|}{v_0 t} - \frac{y}{v_0 t \operatorname{tg}\alpha}\right)H(y)$$

Then we have

$$\varphi_1(t, x, r, \psi) = -\frac{v_0\gamma}{2\pi}\iint_{\Delta}\frac{\operatorname{sgn}\xi\, d\xi\, d\eta}{R} \quad (x \geqslant 0)$$

where the domain of integration Δ is found from the solution of the system of inequalities

$$F[t - (R/c_0), \xi, \eta] > 0, \quad t - (R/c_0) > 0$$

In the special case when the velocity of motion of the edge along the free surface (point A_1) along the Oy axis in Fig.4 is subsonic, i.e. $v_0 \operatorname{tg}\alpha < c_0$, we obtain the following relation for the pressure term in formula (2.4):

$$p_1(t, x, r, \psi) = \frac{\gamma\rho_0 v_0 c_0}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\operatorname{sgn}\xi H(\eta_0)\, d\xi}{R_0/a + \eta_0 - y} \quad x \geqslant 0 \tag{2.5}$$

$$\eta_0 = y + (b - a\{b^2 + (1 - a^2)[(x - \xi)^2 + z^2]\}^{1/2})/(1 - a^2),\ R_0 = R|_{\eta=\eta_0}$$
$$b = (v_0 t - |\xi|)\operatorname{tg}\alpha - y,\ a = v_0 \operatorname{tg}\alpha / c_0 = M \operatorname{tg}\alpha < 1$$
$$M = v_0/c_0, \quad y = r\cos\psi, \quad z = r\sin\psi, \quad x \geqslant 0$$

Transforming the denominator in the integrand in (2.5) using the relation $\eta_0 - y = b - aR_0$ and changing to dimensionless variables

$$x^1 = \frac{x}{v_0 t}, \quad y^1 = \frac{y}{v_0 t}, \quad z^1 = \frac{z}{v_0 t}, \quad \xi^1 = \frac{\xi}{v_0 t}, \quad \eta_0{}^1 = \frac{\eta_0}{v_0 t}$$

we obtain

$$p_1(t, x, r, \psi) \equiv p_1(x^1, r^1, \psi) =$$
$$\frac{\rho_0 v_0 c_0 a\gamma}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\operatorname{sgn}\xi^1 H(\eta_0{}^1)\, d\xi^1}{\{[(1 - |\xi^1|)\operatorname{tg}\alpha - y^1]^2 + (1 - a^2)[(x^1 - \xi^1)^2 + (z^1)^2]\}^{1/2}} \tag{2.6}$$

$$x^1 \geqslant 0,\ y^1 = r^1\cos\psi,\ z^1 = r^1\sin\psi,\ r^1 = [(y^1)^2 + (z^1)^2]^{1/2}$$
$$\eta_0{}^1 = y^1 + (b^1 - a\{(b^1)^2 + (1 - a^2)[(x^1 - \xi^1)^2 + (z^1)^2]\}^{1/2})/(1 - a^2)$$
$$b^1 = (1 - |\xi^1|)\operatorname{tg}\alpha - y^1$$

From now on, we shall omit the superscript 1.

Let us now consider, one after the other, the subsonic case $(M < 1)$ and the supersonic case $(M > 1)$ case.

3. *Penetration of a body with triangular fins at a constant subsonic velocity.*

Let $M < 1$ (and $a < 1$). We shall replace the infinite limits of integration in (2.6) by the finite limits ξ^+ and ξ^-, which can be found from the inequality $\eta_0 > 0$, and have the following form for the points in question belonging to the region $x^2 + y^2 + z^2 < M^{-2}$, $x \geqslant 0$ when $M < 1,\ a < 1$:

$$(1 - M^2)\,\xi^\pm = -xM^2 \pm 1 \mp [(xM^2 \mp 1)^2 + M^2(1 - M^2)(x^2 + y^2 + z^2 - M^{-2})]^{1/2} \quad (\xi^+ > 0,\ \xi^- < 0) \tag{3.1}$$

Since $\xi^-(-x) = \xi^+(x)$, it follows that relation (2.6) can finally be written in the form

$$p_1(x, r, \psi) = (2\pi)^{-1}\rho_0 v_0 c_0 a\gamma\,[A(x) - A(-x)],\ x \geqslant 0 \tag{3.2}$$
$$A(x) = N^{-1}\ln B(x, y, z, \psi), \quad B(x, y, z, \psi) = C_+(x, y, z, \psi)(1 - a^2)\cdot$$
$$(1 + NM)^{-1}\,[N\{(\operatorname{tg}\alpha - y)^2 + (1 - a^2)(x^2 + z^2)\}^{1/2} - \operatorname{tg}^2\alpha + y\operatorname{tg}\alpha - x(1 - a^2)]^{-1}$$
$$C_\pm(x, y, z, \psi) = 1 - x + y(1 - M^2)\operatorname{tg}\alpha \pm N\{(1 - x)^2 + (1 - M^2)\cdot(y^2 + z^2)\}^{1/2},\ N = (\sec^2\alpha - a^2)^{1/2}$$

Thus in accordance with (2.4) and (3.2) we obtain the following expression for the dimensionless pressure at any point (x, r, ψ) of perturbed region $x^2 + y^2 + z^2 < M^{-2}$ $(x \geqslant 0)$:

$$p^\circ(x, r, \psi) = \frac{p - p_0}{\rho_0 v_0 c_0} = \frac{a\gamma}{2\pi N}\ln\prod_{k=1}^{m}\frac{B(x, y_k, z_k, \psi)}{B(-x, y_k, z_k, \psi)} \tag{3.3}$$
$$y_k = r\cos[\psi - 2\pi(k - 1)m^{-1}],\ z_k = r\sin[\psi - 2\pi(k - 1)m^{-1}]$$

Formulas (3.2) and (3.3) together yield the following results:

1°. When $x \to 0$, we have $B(+0, y_k, z_k, \psi) = B(-0, y_k, z_k, \psi)$, and this yields $p^\circ \to 0$.

2°. When $x^2 + y^2 + z^2 \to M^{-2}$, we have $B(\pm x, y_k, z_k, \psi) \to 1$ and $p^\circ \to 0$.

3°. When $z_k = 0,\ 0 < x < 1$ and $y_k \to (1 - x)\operatorname{tg}\alpha$, we find that the demoninator in the expression for $B(x, y_k, z_k, \psi)$ tends to zero. Therefore

$$\ln[B(x, y_k, z_k, \psi)/B(-x, y_k, z_k, \psi)] \to +\infty$$

and p^0 has a logarithmic singularity at the corresponding edge of the fin.

4°. At the points of the x axis outside the point B, i.e. when $y_k = z_k = 0,\ x \neq 1$, the pressure p^0 is finite, but has a logarithmic singularity when $x \to 1 \pm 0$.

The above results satisfy the constraints imposed on the order of the singularities shown above; solution (3.3) is therefore unique.

We note that although the velocity component $\partial_r \varphi_1$ in the auxiliary solution φ_1 has a logarithmic singularity on the OB axis (Fig.4), the singularities cancel each other in the course of constructing the required solution φ, so that we have, in accordance with /4/,

$$\sum_{k=1}^{m} \cos\left[\psi - 2\pi (k-1) m^{-1}\right] \equiv 0$$

and the velocity at the axis OB outside the neighbourhoods of the points O and B is limited.

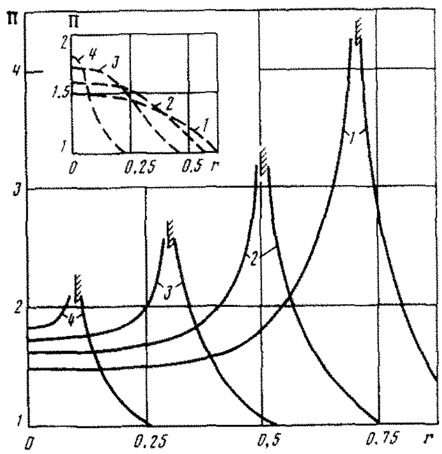

Fig.5

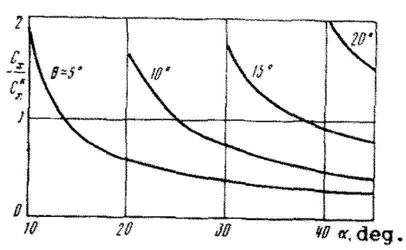

Fig.6

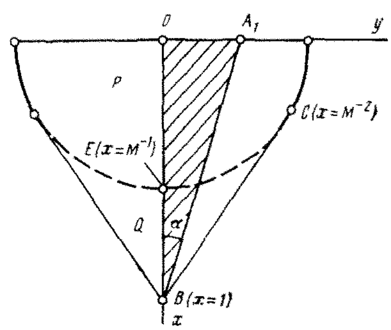

Fig.7

A detailed investigation of the flow characteristics was carried out for a star-shaped body with fins of triangular shape and cross-section (Fig.4), with a number of vanes equal to $m = 4$. Fig.5 shows data on the pressure distribution over the fin surface $\psi = 0$ (the solid lines) and in the plane $\psi = \pi/4$, passing through the bisectrix of the angle between the fins, in various cross-sections $x = \mathrm{const}$ (the dashed lines). Curves 1-4 correspond to $x = 0.3,\ 0.5,\ 0.7,\ 0.9$.

The computations were carried out for $M = 0.8,\ \alpha = \pi/4$, and the pressure was referred, for every cross-section $x = \mathrm{const}$, to the mean value of the pressure at the wall of a two-fin wedge and denoted by Π. An increase of almost a factor of 2 in the pressure over the whole region of perturbed flow between the fins is due to their strong interference. The pressure retains its high values at the wall and along any ray within the region bounded by the fins, and drops substantially outside this region.

Thus, analysing the pressure distribution we can draw qualitative conclusions that the penetration of a star-shaped body into a liquid is accompanied by the formation of a wide region of high pressure which moves together with the body. A similar assertion was deduced in /1/ for an ensemble of wedges. The dependence of the resistance of the star-shaped body referred to the resistance of a circular cone of equivalent length and volume, with an aperture half-angle θ, on the angle α at the tip of the fin, is shown in Fig.6 for $M = 0.9$. We see that the resistance of a star-shaped body can be greater than the resistance of the equivalent cone (when the angle α) is relatively small), and smaller when the angle α. increases.

The drop in resistance was noted earlier in the case of hypersonic gas flow past a star-shaped solid. This was explained by the fact that in the case of a body of constant length and volume $(\theta = \text{const})$ the increase in the tip angle α results in making the fin thinner and reducing the wave resistance of the body. However, in the case of hypersonic flow past a star-shaped body the wave resistance is smaller than that of the equivalent cone over a wide range of variation in the value of α. The increase in the resistance of such a body entering the liquid with subsonic velocity, compared with the resistance of the equivalent cone, is apparently caused by interference from the neighbouring fins, which more or less ceases when the flow is hypersonic.

4. Penetration of a star-shaped body with a constant supersonic velocity.

Let $M > 1$ $(a < 1)$. Just as in the subsonic case $(M < 1, a < 1)$, discussed above, the solution $p_1(x, r, \psi)$ of the auxiliary problem for the constant velocity of entry v_0 and a triangular fin of cross-section S_1 is written in the form of the integral (2.6). In the subsonic case the solution was written in terms of elementary functions. We shall use the same methods to reduce the integral (2.6) to elementary functions in the present, supersonic case of entry with subsonic edges (Fig.7).

Just as in the subsonic case, the domain of integration in ξ is determined by the condition $\eta_0 > 0$, where η_0 is given by the formula in (2.6) (the superscript 1 on the variables has been omitted everywhere from formula (2.6) onwards). This condition leads to the system

$$a^2\{b^2 + (1 - a^2)[(x - \xi)^2 + z^2]\} < [b + y(1 - a^2)]^2$$
$$b + y(1 - a^2) > 0 \tag{4.1}$$

The first inequality in (4.1) to the regions $\xi \geqslant 0$ and $\xi < 0$ reduces to quadratic inequalities for ξ, which can conveniently be written in the form

$$u^2(M^2 - 1) \pm 2(1 \mp x)u + M^2(y^2 + z^2) - (1 \mp x)^2 < 0 \quad (u \equiv \xi - x) \tag{4.2}$$

where the upper sign is taken for $\xi \geqslant 0$ and the lower sign for $\xi < 0$. The second inequality in (4.1) takes the form

$$-1 + yMa < \xi < 1 - yMa$$

Before obtaining the solution of system (4.1) in various regions of perturbed motion of the medium, we shall write expressions for the roots of the quadratic trinomials on the left-hand sides of inequalities (4.2). These have the form

$$\xi_i^{\pm} = \{xM^2 \mp 1 \mp (-1)^i[(xM^2 \mp 1)^2 + M^2(1 - M^2)(x^2 + y^2 + z^2 - M^{-2})]^{1/2}\} \times (M^2 - 1)^{-1} \quad (i = 1, 2) \tag{4.3}$$

where $\xi_{1,2}^{+}$ $(\xi_{1,2}^{-})$ are the roots of the quadratic trinomial from the left-hand side of (4.2) for the upper (lower) signs in (4.2).

Depending on the values of the variables and parameters, these roots may be real or complex, and they may appear, or not appear in the regions $\xi \geqslant 0$ and $\xi < 0$.

In the general case we have six regions of perturbed motion appearing when the body enters the liquid at supersonic velocity. The regions are symmetrical and occupy the whole half-space $x > 0$ of the space in question.

Analysis shows that when $M > 1, a < 1$, the solution of the auxiliary problem is non-zero only in regions P and Q (see the cross-section $z = 0$ or the perturbed region in Fig.7), and this agrees with physical ideas. As a result we obtain the following relations for the limits of integration:

$$\xi_1^- < \xi < \xi_1^+ \text{ in the region } P\ (x^2 + y^2 + z^2 < M^{-2},\ x > 0)$$
$$\xi_2^+ < \xi < \xi_1^+ \text{ in the region } Q\ (x^2 + y^2 + z^2 > M^{-2},$$
$$M^{-2} < x < 1 - [(z^2 + y^2)(M^2 - 1)]^{1/2})$$

For region P the limits of integration ξ_1^- and ξ_1^+ are the same as in the subsonic case. For region Q, where the roots ξ_2^+ and ξ_1^+, serve as the limits of integration, a solution can also be found using the method of deriving the expressions (3.2) and (3.3). As a result, using (2.4) we obtain the following expressions for the dimensionless pressure analogous to (3.3) in the region Q:

$$p^\circ(x, r, \psi) = \frac{p - p_0}{\rho_0 v_0 c_0} = \frac{a\gamma}{2\pi N} \ln \prod_{k=1}^{m} \frac{C_+(x, y_k, z_k, \psi)}{C_-(x, y_k, z_k, \psi)} \qquad (4.4)$$

(the notation used is the same as that in formulas (3.2)).

Thus in the case of supersonic entry of a star-shaped body $(M > 1)$ with subsonic edges $(a = M \operatorname{tg} \alpha < 1)$, the formulas desribing the solution in the subsonic region $x^2 + y^2 + z^2 < M^{-2}$ (region P in Fig.7), are identical with formulas (3.3) obtained for subsonic penetration. In the supersonic region the solution is non-zero only within the characteristic cone $(M^2 - 1)^{1/2}(y^2 + z^2)^{1/2} < 1 - x$, bounded from above by the sphere $x^2 + y^2 + z^2 = M^{-2}$ (region Q in Fig.7). The solution for the dimensionless pressure within this region is given by formulas (4.4).

Analysis the solution we find that at the edges of the body (BA_1 in Fig.7) the pressure has a logarithmic singularity in both the supersonic and subsonic regions. When the tip of the body (B in Fig.7) is approached from different directions, the pressure takes various finite values, so that the function p° has no limit at the tip of the body. On the axis of the body (OB in Fig.7) the pressure is continuous and bounded everywhere, and in the supersonic region (BE in Fig.7) it has a constant value equal to $p^\circ = (a\gamma m/(2\pi N)) \ln [(1 + N)/(1 - N)]$.

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